Lectures in International Monetary Economics

Paolo Canofari
University of Teramo and University of Rome “Tor Vergata”
pcanofari@unite.it

October 2013

The Basic Monetary Model of the Exchange Rate

The basic monetary model of exchange rate determination is a useful starting point to analyze nominal exchange rate dynamics and serves as a building block for all the models we present in this book.

We base our discussion of the monetary model on the analytical framework of intertemporal optimization. This has two major advantages: (i) the model is build on firm microeconomic foundations, which means that the behavioral relationships are derived from the intertemporal optimal choice of agents; (ii) substantial issues related to model dynamics and optimal money growth or exchange rate depreciation can be addressed in a more natural and convenient way.

The monetary approach to exchange rate dates back to the sixties. The monetary approach, pioneered by Robert Mundell (1968) and Harry Johnson (1972), emerged as the dominant exchange rate model in the 1970s, see Frenkel and Johnson (1978). In the mid-1970s and early 1980s it was extended to include rational expectations and short term overshooting (Dornbusch, 1976) and to allow for multiple traded goods and real shocks across countries (Stockman, 1980; Lucas, 1982).

Section 1.1 describes the basic structure of the model. The economy is small and agents are assumed to consume a single good and to hold two assets, domestic money and traded bonds. The focus is on the typical household's maximization problem and on the evolution of government debt and the current account of the domestic economy. Section 1.2 focuses on the perfect foresight equilibrium and on the dynamics of the model. Section 1.3 shows how a solution path for the nominal exchange rate can be found and examines the stability properties of the stationary point. Section 1.4 compares the effects of anticipated and unanticipated changes in the exogenous variables. Section 1.5 discusses the optimal monetary growth policy or exchange rate depreciation.

1.1 The Structure of the Model

We consider a small open economy populated by identical, utility-maximizing households and a government. We assume that households live forever and have perfect foresight. We also assume that household's financial wealth includes only two assets: domestic money and a world traded bond denominated in foreign currency. The production technology is inessential for the present purpose. We therefore focus on an endowment economy with a single, perishable, consumption good. Modifications to this basic setting will be introduced in subsequent chapters. There are no barriers to trade and international capital markets are perfect, so that purchasing power parity and uncovered interest parity hold at all times. Domestic and foreign price levels are linked by the relationship

$$P_t = S_t P_t^* \tag{1.1}$$

where S_t is the nominal exchange rate, defined as the price of foreign currency in terms of home currency, while P_t and P_t^* denote the home and foreign price level, respectively. The domestic nominal interest rate i_t is tied to the (constant) foreign nominal rate i^* by the arbitrage equation

$$i_t = i^* + \frac{\dot{S}_t}{S_t}, \tag{1.2}$$

where $\dot{S}_t \equiv (dS_t/dt)$ and $\dot{S}_t/S_t$ is the instantaneous expected rate of change in S_t, which, given perfect foresight, equals the actual rate[1].

For notational convenience we now set the number of identical households in the economy equal to 1, focusing on just one unit's, or representative household's behavior. We then assume that both consumption and real money balances enter the utility function. The basic idea is that money, by allowing agents to save time in conducting their transactions, yields a direct utility that is not connected to other assets such as bonds. The devise of putting money directly into the utility function was first used in Sidrauski (1967) and Brock (1974, 1975) and is now a standard approach in intertemporal optimization modelling[2]. Its purpose is to capture the fundamental role money plays in the economy as a store of value, a medium of exchange and a unit of account. Within the infinite horizon model another approach has also been used: modelling the process of transaction explicitly through the so-called cash-in-advance constraint originally advocated by Clower (1967)[3]. However, as shown by Feenstra (1986), the two approaches are equivalent if certain regularity condition are satisfied.

[1] Throughout this book we shall use primes to denote total derivatives, appropriate subscripts to denote partial derivatives, and dots above the variable to denote time derivatives. Thus, we shall let:

$f'(x) \equiv \frac{df}{dx}, \qquad f_{x_i}(x_1, ..., x_n) \equiv \frac{\partial f(\cdot)}{\partial x_i}, \qquad \dot{x} \equiv \frac{dx}{dt}.$

[2] See, for example, Feenstra (1986), Turnovsky (1995, 1997), Obstfeld and Rogoff (1996), Walsh (2003).

[3] See, for example, Helpman (1981), Lucas (1982), Svensson(1985), Lucas and Stokey (1987), Grilli and Roubini (1992).

We now derive the solutions for the equilibrium path of the nominal exchange rate and real money balances. Assume, for simplicity, that the utility function has the Cobb-Douglas-Wicksell form

$$U\left(C_t, \frac{M_t}{P_t}\right) = C_t^{\alpha}\left(\frac{M_t}{P_t}\right)^{1-\alpha}, \quad 0 < \alpha < 1.$$

Condition (1.10b) implies

$$\frac{M_t}{P_t} = \eta \frac{C_t}{i_t},$$

where $\eta \equiv (1-\alpha)/\alpha$. This is a standard demand for money equation with consumption rather than income measuring the volume of transactions. We can write the equilibrium condition for the money market as

$$\frac{M_t}{P_t} = F(C_t, i_t), \qquad F_C(C_t, i_t) > 0, \quad F_i(C_t, i_t) < 0. \tag{1.22}$$

Equation (1.22) can be solved for the nominal interest to obtain

$$i_t = i\left(\frac{M_t}{P_t}, C_t\right), \qquad i_{\frac{M}{P}}\left(\frac{M_t}{P_t}, C_t\right) < 0, \quad i_C\left(\frac{M_t}{P_t}, C_t\right) > 0.$$

Combining this equation with the purchasing power parity and uncovered interest parity conditions, we obtain

$$\dot{s}_t \equiv \frac{\dot{S}_t}{S_t} = i\left(\frac{M_t}{S_t P_t^*}, C_t\right) - i^*. \tag{1.23}$$

Equation (1.23) gives a dynamically unstable exchange rate, as illustrated in the phase diagram in Figure 1. The phase line is upward sloping with a slope given by $-i_{\frac{M}{P}}\left(M/S^2P^*\right) > 0$. There is a unique stationary point with $\dot{s} = 0$ shown as $\overline{S}$. To the right of $\overline{S}$, $S_t > \overline{S}$, $\dot{s} > 0$ and the exchange rate depreciates forever. To the left of $\overline{S}$, $S_t < \overline{S}$, $\dot{s} < 0$ and the exchange rate continues to appreciate without end. Hence, unless the economy initially jumps at $\overline{S}$, there is no convergence to the equilibrium point $\overline{S}$.

Suppose that the economy is initially in steady state, M is constant and that there is a small rise in S from $\overline{S}$. The rise in the exchange rate means a higher domestic price level and a lower real quantity of money, which is consistent with equilibrium only if the demand for money decreases. This requires an increase in the domestic interest rate, which could come about only if agents expect a higher rate of depreciation, given the interest parity condition. As the expected (and actual) rate of depreciation goes up, the inflation rate starts to rise and the real stock of money starts to fall further. Money market equilibrium therefore requires a further increase in the interest rate, which in turn implies a further increase in the depreciation rate. The exchange rate continues to depreciate (at an accelerating speed since $\dot{S}/S$ increases), the inflation rate and the nominal

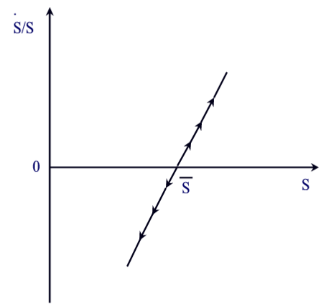

Figure 1: The instability of exchange rate

interest rate increase steadily and the real stock of money tends to zero as time goes towards infinity. The same unstable path woks in reverse for a small drop in S from $\overline{S}$.

The monetary approach rules out explosive paths by assuming that the exchange rate jumps initially to $\overline{S}$ and then stays there forever, unless the system is disturbed in some way. As stressed in Section 1.2, the size of the initial jump is determined by the transversality condition, which requires the present discounted value of the real money stock to be zero at the end of the planning horizon. This in general leads to a bounded nominal exchange rate level[4].

Rewriting equation (1.15) using (1.23), we obtain

$$\frac{d(M_t/P_t)/dt}{(M_t/P_t)} = \Psi\left(\frac{M_t}{P_t}, C_t\right) \equiv \mu - \dot{p}^* - \left[i\left(\frac{M_t}{P_t}, C_t\right) - i^*\right]. \tag{1.24}$$

The partial derivatives of the function Ψ are

$$\Psi_{\frac{M}{P}} = -i_{\frac{M}{P}} > 0 \quad \text{and} \quad \Psi_C = -i_C < 0.$$

Equation (1.24) describes a dynamic unstable equation for each fixed C, since $\Psi_{\frac{M}{P}} > 0$ as illustrated in figure 2, where $\left(\overline{M/P}\right)$, defined by

$$\left[i\left(\frac{\overline{M}}{P}, \overline{C}\right) - i^*\right] \equiv \dot{s} = \mu - \dot{p}^*,$$

[4] Restrictions on the underlying utility functions needed to preclude explosive paths are discussed at length in Brock (1975), Calvo (1978a), Gray (1984), Obstfeld and Rogoff (1983), Obstfeld (1984a).

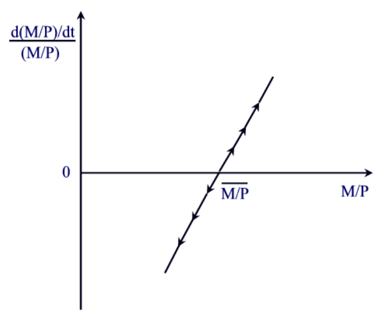

Figure 2: The instability of real money balances

is the unique steady-state level of real balances. Unless the value of (M/P) jumps initially to $\left(\overline{M/P}\right)$, real balances diverge away from the steady state. When $(M/P) = \left(\overline{M/P}\right)$, $\dot{s} = \mu - \dot{p}^*$.

The non-explosive solution to equation (1.23) depends on the entire future time path of all the exogenous variables of the model. The solution for the current nominal exchange rate is entirely *forward looking*. Loglinearizing equation (1.22)

$$m_t - p_t = \alpha_1 c_t - \alpha_2 i_t,$$

where $m_t \equiv \ln M_t$, $p_t \equiv \ln P_t$, $c_t \equiv \ln C_t$, and α_1 and α_2 are positive constants, using (1.2) to eliminate the domestic interest rate, taking logs of (1.1) to substitute for p_t, and rearranging terms, we obtain the following differential equation for the nominal exchange rate

$$\dot{s}_t = \frac{1}{\alpha_2} s_t - \frac{1}{\alpha_2}\omega_t, \tag{1.25}$$

where $s_t \equiv \ln S_t$, $\omega_t \equiv m_t - p_t^* - \alpha_1 c_t + \alpha_2 i^*$, and $p_t^* \equiv \ln P_t^*$. A solution to this equation is

$$s_t = e^{\frac{t}{\alpha_2}} \left[s_0 - \frac{1}{\alpha_2} \int_0^t \omega_v e^{-\frac{v}{\alpha_2}} dv \right], \tag{1.26}$$

or equivalently,

$$s_t = [s_0 - F_0]\, e^{\frac{t}{\alpha_2}} + F_t, \tag{1.26'}$$

where s_0 is the initial exchange rate, $F_t \equiv \frac{1}{\alpha_2} \int_t^\infty \omega_v e^{-\frac{(v-t)}{\alpha_2}} dv$ is the *fundamental* or *equilibrium solution*, and $[s_0 - F_0]\, e^{\frac{t}{\alpha_2}}$ is the *bubble* component capturing possible deviations from F_t unrelated to the underlying path of the exogenous variables or "fundamentals" of the economy, that is paths where the exchange rate explodes only because it is expected to do so [Sargent and Wallace (1973), Flood and Garber (1980)][5]. Equation (1.26) [or (1.26')] suggests that the nominal exchange rate goes to plus or minus infinity as $t \to \infty$ unless the term in brackets is zero. Therefore, in order to rule out such explosive paths for s_t, we need to impose the restriction

$$s_0 = \frac{1}{\alpha_2} \lim_{t \to \infty} \int_0^t \omega_v e^{-\frac{v}{\alpha_2}} dv = \frac{1}{\alpha_2} \int_0^\infty \omega_v e^{-\frac{v}{\alpha_2}} dv.$$

Substituting for s_0 into (1.26) yields

$$s_t = F_t \equiv \frac{1}{\alpha_2} \int_t^\infty \omega_v e^{-\frac{(v-t)}{\alpha_2}} dv. \tag{1.27}$$

The current nominal exchange rate is a function of the expected time path of all the variables (or fundamentals) included in ω_t from t until forever. The discount rate is the inverse of the semi-elasticity of money demand to the interest rate. The fundamental solution (1.27) implicitly assumes that

$$\lim_{v \longrightarrow \infty} \omega_v e^{-\frac{(v-t)}{\alpha_2}} = 0. \tag{1.28}$$

This requires the growth rate of ω be below $1/\alpha_2$ in absolute value. Condition (1.28) is only necessary but not sufficient for boundedness, since s_t may follow an explosive path even if (128) is met. An example is given the next section.

The unique solution excluding bubble paths implies that the economy is always on its *saddle path* and characterizes the equilibrium dynamics of the system as saddle-point stable[6].

1.4 Permanent and Temporary Changes in the Exogenous Variables and Exchange-Rate Dynamics

A unique stationary value for the nominal exchange rate exists only when the exogenous variables included in the *forcing term* ω_t are expected to remain

[5] The general (forward looking) solution to (1.25) is

$$s_t = ke^{\frac{t}{\alpha_2}} + F_t,$$

where k is an arbitrary constant chosen to satisfy some condition known as *initial* or *boundary condition.* Equation (1.25) has an infinity of solutions unless an appropriate value for k is selected. For example, if we choose an initial value for s_t by setting $t = 0$ in the above expression, then $k = s_0 - F_0$ and the solution will be as in (1.26').

[6] Equation (1.25) has an infinity of unstable solutions associated with the positive root $1/\alpha_2$. For a given sequence of ω_t there is a unique value of the exchange rate for which s_t does not explode. Such value is identified by choosing the particular solution with initial condition $k \equiv s_0 - F_0 = 0$. It is the only solution that places the economy on its saddle-path equilibrium.

constant over the entire future time path. Changes in ω_t alter the time path of s_t in this model, where dynamics is entirely *exogenous*. Denote the initial expected time path for the forcing term as

$$\omega_v^0, \quad \forall\ v \geq t.$$

Suppose that after a once-and-for-all change in ω_t, the new time path for ω_t is expected to be

$$\omega_v^1 \equiv \omega_v^0 + \xi, \quad \forall\ v \geq t, \quad \xi > 0,$$

where ξ is the exogenous change in ω_t. The solution for the nominal exchange rate is

$$s_t = \frac{1}{\alpha_2} \int_t^\infty \left[\omega_v^0 + \xi\right] e^{-\frac{(v-t)}{\alpha_2}} dv = s_t^0 + \frac{1}{\alpha_2} \int_t^\infty \xi e^{-\frac{(v-t)}{\alpha_2}} dv = s_t^0 + \xi, \quad (1.29)$$

where

$$s_t^0 = \frac{1}{\alpha_2} \int_t^\infty \omega_v^0 e^{-\frac{(v-t)}{\alpha_2}} dv$$

denotes the time path of the nominal exchange rate before the change in ω_t. The exchange rate jumps immediately in response to the change in the forcing term.

We now focus on the change in ω_t caused by an increase in the money supply, assuming that all other exogenous variables remain fixed. Suppose that the initial time path of the money supply was

$$m_t = m_0, \quad \forall\ t \geq t_0,$$

where m_0 is a constant. After an unanticipated and once-and-for-all increase in m_t that leaves $\dot{m}_t$ unchanged, the new time path for the money supply is expected to be

$$m_t = m_0 + \xi, \quad \forall\ t \geq t_0, \quad \xi > 0.$$

From (1.29), the solution for the exchange rate is

$$s_t = s_0 + \xi, \quad (1.30)$$

where

$$s_0 = \frac{1}{\alpha_2} \int_t^\infty m_0 e^{-\frac{(v-t)}{\alpha_2}} dv = m_0.$$

The solution (1.30) shows that the jump in the money stock at $t = t_0$ leads to an immediate jump in the nominal exchange rate of the same magnitude. The domestic price level also jumps by the same factor ξ. As a result, the real money stock and all the real variables in the economy remain at their steady-state equilibrium values, and the system shows no transitional dynamics. The path of the nominal money stock, the exchange rate, the price level and real money balances are described in figure 3, where $m_1 - m_0 = s_1 - s_0 = p_1 - p_0 = \xi$.

The economic intuition of this result is simple. Forward-looking agents know that the initial increase in the money supply will be permanent, and with prices

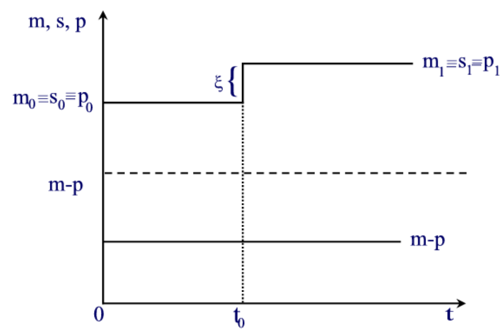

Figure 3: Effects of unanticipated monetary expansion

fully free-flexible and reflecting all the available information, the exchange rate and the price level must fully respond to the change in m.

Now consider the response of the system to an announcement at $t = t_0$ that the money stock will increase by ξ at time t_1 in the future. The anticipated time profile for m_t is

$$m_t = m_0, \quad \forall\, t \leq t_1 \tag{1.31a}$$
$$m_t = m_0 + \xi, \quad \forall\, t \geq t_1. \tag{1.31b}$$

Substituting (1.31) into (1.29), we find that the solution for the nominal exchange rate is

$$s_t = \frac{1}{\alpha_2}\int_t^{t_1} m_0 e^{-\frac{(v-t)}{\alpha_2}} dv + \frac{1}{\alpha_2}\int_{t_1}^{\infty} (m_0+\xi)\, e^{-\frac{(v-t)}{\alpha_2}} dv \Longrightarrow$$
$$s_t = m_0 + \xi e^{-\frac{(t_1-t)}{\alpha_2}}, \qquad \forall\;\; t_0 \leq t \leq t_1, \tag{1.32a}$$

and

$$s_t = \frac{1}{\alpha_2}\int_t^{\infty} (m_0+\xi)\, e^{-\frac{(v-t)}{\alpha_2}} dv = m_0 + \xi, \quad \forall\;\; t \geq t_1. \tag{1.32b}$$

The time path for the nominal exchange rate is drawn in figure 4. It shows that s_t jumps by $\xi e^{-\frac{(t_1-t)}{\alpha_2}}$ at $t = t_0$ and then rises smoothly until time t_1, when the new steady-state value is achieved. No jump occurs at $t = t_1$, when the increase in the money stock takes place.

The announcement of the future expansion in the money supply causes the exchange rate (and the domestic price) level to jump immediately at time t_0.

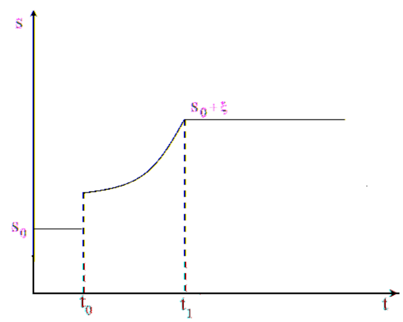

Figure 4: Effects of anticipated monetary expansion

This is because individuals anticipate the future depreciation in the exchange rate at $t = t_1$. The increase in s by the amount $\xi e^{-\frac{(t_1-t)}{\alpha_2}}$ at $t = t_0$ therefore discounts to the present the effects of the expected future monetary expansion. The more distant in the future is the expansion, the smaller is the current response in the exchange rate. The exchange rate depreciates at an accelerating speed between t_0 and t_1, causing the domestic interest rate to rise in order to preserve equilibrium in the money market where the real money supply is reduced due to the jump in the price level. At time t_1, when the monetary expansion takes place, the rate of depreciation falls back to zero, the domestic interest rate equals again the foreign rate i^*, and the real money stock is brought back to its original level.

The time path for s_t is continuous at time $t = t_1$, hence the exchange rate does not jump when m_t changes but at $t = t_0$, when the news of the future expansion first arrives. However, the expected rate of depreciation $\dot{s}_t$ does jump at t_1. From (1.32a) and (1.32b)

$$\frac{ds_t}{dt} = \frac{\xi}{\alpha_2} e^{-\frac{(t_1-t)}{\alpha_2}}, \qquad \forall\ t_0 \leq t \leq t_1,$$

and

$$\frac{ds_t}{dt} = 0, \qquad \forall\ t \geq t_1.$$

At $t = t_1$ the left-hand time derivative of the exchange rate exceeds the right-hand time derivative by $\xi/\alpha_2 > 0$. The fall of $\dot{s}_t$ to zero at time t_1 induces a fall in the home interest rate, that increases the demand for money to compensate for the jump in the money supply at time t_1. Money-market equilibrium is maintained and there is no need for the exchange rate to jump.

Consider now the response of the system to a jump in the money stock that is known to be temporary. The time profile of the money supply is now

$$\begin{aligned} m_t &= m_0 + \xi, \qquad \forall\ t_0 \leq t \leq t_1 \\ m_t &= m_0, \qquad \forall\ t \geq t_1, \end{aligned}$$

and the solution for s_t is

$$s_t = \frac{1}{\alpha_2} \int_t^{t_1} (m_0 + \xi)\, e^{-\frac{(v-t)}{\alpha_2}} dv \ + \frac{1}{\alpha_2} \int_{t_1}^{\infty} m_0 e^{-\frac{(v-t)}{\alpha_2}} dv \Longrightarrow$$

$$s_t = m_0 + \xi \left[1 - e^{-\frac{(t_1 - t)}{\alpha_2}} \right], \qquad \forall\ t_0 \leq t \leq t_1, \tag{1.33a}$$

$$s_t = \frac{1}{\alpha_2} \int_t^{\infty} m_0 e^{-\frac{(v-t)}{\alpha_2}} dv = m_0, \qquad \forall\ t \geq t_1. \tag{1.33b}$$

Figure 5 displays the time path of the nominal exchange rate and shows the two phases of adjustment. At $t = t_0$, s_t jumps upward, but less than it would have done if the increase in m_t had been permanent. The exchange rate falls (i.e. appreciates) until time $t = t_1$, when the change in the money stock is reversed and the initial equilibrium level of s_t is restored.

The intuition underlying this result is the following. Forward-looking agents know that at time $t = t_1$ both the exchange rate and the real money balances will be back at their initial equilibrium levels. Before the increase in m is reversed, the real money stock is higher and the real demand for money must be higher. This requires a lower interest rate, which is possible only if the exchange rate is expected to appreciate. Thus, immediately before t_1, s_t must be falling and above its steady-state equilibrium value. The exchange rate must first jump upwards at t_0 and then appreciate gradually towards its old level s_0.

We have considered so far exogenous shocks where the level but not the growth rate of the money supply has been modified. Assume instead that at time $t = t_0$ the monetary authorities change the growth rate of m from 0 to a constant rate $\mu > 0$. The time profile for the money supply is now

$$m_t = m_0 + \mu t, \qquad \forall\ t \geq t_0, \tag{1.34}$$

where m_0 denotes the initial expected time path of the money supply. Substituting (1.34) into (1.29), gives

$$\begin{aligned} s_t = \frac{1}{\alpha_2} \int_t^{\infty} (m_0 + \mu v)\, e^{-\frac{(v-t)}{\alpha_2}} dv = \frac{1}{\alpha_2} \int_t^{\infty} m_0 e^{-\frac{(v-t)}{\alpha_2}} dv + \\ \frac{\mu}{\alpha_2} e^{\frac{t}{\alpha_2}} \int_t^{\infty} v e^{-\frac{v}{\alpha_2}} dv = m_0 + \frac{\mu}{\alpha_2} e^{\frac{t}{\alpha_2}} \int_t^{\infty} v e^{-\frac{v}{\alpha_2}} dv. \end{aligned}$$

Integrating by parts, we obtain

$$\int_t^{\infty} v e^{-\frac{v}{\alpha_2}} dv = \left[-\alpha_2 v e^{-\frac{v}{\alpha_2}} \right]_t^{\infty} + \alpha_2 \int_t^{\infty} e^{-\frac{v}{\alpha_2}} dv = \alpha_2 (t + \alpha_2)\, e^{-\frac{t}{\alpha_2}}.$$

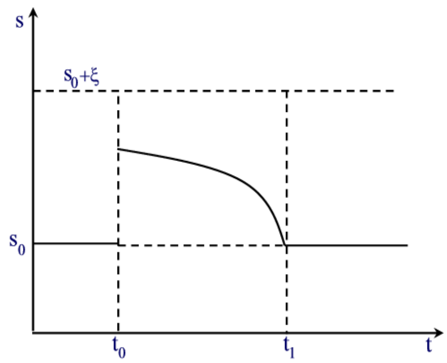

Figure 5: Effects of a temporary monetary expansion

The solution for s_t is

$$s_t = m_0 + \mu t + \mu \alpha_2 = s_0 + \mu \left(t + \alpha_2 \right), \tag{1.35}$$

hence

$$\dot{s}_t = \mu. \tag{1.36}$$

Equations (1.35) and (1.36) reveal that the exchange rate jumps upwards by the amount $\mu\alpha_2$ at time t_0, depreciating steadily at the same constant rate of the nominal money growth thereafter. The rate of depreciation and the real money supply remain constant, as one should expect with a nominal money supply expanding at a constant rate.

Consider now the response of the economy to the announcement of a permanent future increase in the growth rate of the nominal quantity of money. Suppose that the monetary authorities announce at time $t = t_0$ that m_t will follow the path

$$\begin{aligned} m_t &= m_0, & \forall\, t \leq t_1 \\ m_t &= m_0 + \mu t & \forall\, t \geq t_1, \end{aligned}$$

so the money growth rate is expected to jump at time t_1 from 0 to $\mu > 0$. The solutions are

$$\begin{aligned} s_t &= m_0 + \mu\,(t_1+\alpha_2)\,e^{-\frac{(t_1-t)}{\alpha_2}} \quad \forall\ t_0 \le t \le t_1, && (1.37a)\\ \dot{s}_t &= \frac{\mu}{\alpha_2}\,(t_1+\alpha_2)\,e^{-\frac{(t_1-t)}{\alpha_2}} \quad \forall\ t_0 \le t \le t_1, && (1.37b)\\ s_t &= m_0 + \mu\,(t+\alpha_2) \quad \forall\ t \ge t_1, && (1.38a)\\ \dot{s}_t &= \mu \quad \forall\ t \ge t_1\ . && (1.38b) \end{aligned}$$

At time $t = t_0$ the exchange rate jumps by $\mu\,(t_1+\alpha_2)\,e^{-\frac{(t_1-t_0)}{\alpha_2}}$, since agents discount to the present the effects of the future increase in money growth. The exchange rate then depreciates along the path towards t_1 according to the exponential function $(\mu/\alpha_2)\,(t_1+\alpha_2)\,e^{-\frac{(t_1-t_0)}{\alpha_2}}$. At time $t = t_1$, the anticipated monetary expansion actually occurs, the exchange rate level is at $m_0 + \mu\,(t_1+\alpha_2)$ and the rate of depreciation drops sharply from $[(\mu/\alpha_2)\,t+\mu]$ to μ. Thereafter, s_t keeps on depreciating at the constant monetary growth rate μ.